Homework-4

1. Suppose A is a normal subgroup of G and A is abelian. Also, suppose $AH = G$ for some subgroup H. Show that $A \cap H$ is a normal subgroup of G.

2. Let H be a subgroup of a group G. The normalizer of H in G is defined as $N_G(H) = \{g \in G : gHg^{-1} = H\}$ and the centralizer of H in G is defined as $C_G(H) = \{g \in G : gh = hg \text{ for all } h \in H\}$.

Show that $N_G(H)/C_G(H)$ is isomorphic to a subgroup of $Aut(H)$.

3. Prove that every finite group having more than two elements has a non-trivial automorphism.

4. We proved in the class that every nontrivial finitely generated group has a maximal subgroup. Give an example to show that "finitely generated" cannot be dropped from the assumption to show the existence of a maximal subgroup.