

Homework-3

1. Does there exist a group G such that $G/Z(G)$ is isomorphic to a direct product of two groups A and B where A is a cyclic group of order 3 and B is a cyclic group of order 5?
2. Let G be a finite group and H a subgroup of G of order n . If H is the only subgroup of G of order n , then show that H must be normal in G .
3. If a cyclic subgroup H of G is normal in G , then show that every subgroup of H is normal in G .
4. Let G be a finite group and N be a normal subgroup of G with $[G : N] = k$. Prove that if x is any element of G such that order of x is relatively prime to k , then x must lie in N .
5. Let G be a finite group and suppose the automorphism φ of G sends more than three-quarters of the elements of G onto their inverses. Prove that $\varphi(x) = x^{-1}$ for all $x \in G$ and hence conclude that G is abelian.