Homework-1

1. Let G be a group such that $|G| \leq 5$. Prove that G must be abelian.

2. Let G be a group with the property that there do not exist three elements $x, y, z \in G$, no two of which commute. Prove that G must be an abelian group.

3. Let S be a subset of a finite group G with $|S| > \frac{|G|}{2}$. If the set S^2 is defined as $S^2 = \{xy : x, y \in S\}$, then prove that $S^2 = G$.

4. Describe D_4 , the group of symmetries of a square and write its multiplication table.