

## Homework 9

1. Prove that for every prime power  $p^n$ , a field of order  $p^n$  exists.
2. Prove that in an abelian group  $G$ , if there are elements  $a$  and  $b$  of order  $m$  and  $n$ , respectively, then there is an element  $c$  in  $G$  such that order of  $c$  is the least common multiple of  $m$  and  $n$ .
3. Give example of a group with two elements of finite order such that their product has infinite order.
4. Let  $F$  be a finite field of order  $p^n$ . Prove that the Galois group  $Gal(F/F_p)$  is a cyclic group of order  $n$ .