

1. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be defined by $T(x_1, x_2) = (x_1 - x_2, x_1, 2x_1 + x_2)$.

(a) Show that T is a linear map.

(b) Find dimensions of $\text{null}(T)$ and $\text{Im}(T)$.

(c) Write matrix representation of T with respect to standard bases of \mathbb{R}^2 and \mathbb{R}^3 .

(d) Write matrix representation of T with respect to bases $\{(1, 2), (0, 1)\}$ of \mathbb{R}^2 and $\{(1, 1, 0), (0, 1, 1), (2, 2, 3)\}$ of \mathbb{R}^3 .