Homework 7

1. Let $T : M_{2\times 2}(\mathbb{R}) \to \mathbb{R}$ be the map defined as $T(\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}) = a_{11} + a_{22}$.

(a) Show that T is a linear transformation.

(b) Find the matrix representation of T with respect to ordered basis $B_1 = \{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\}$ of $M_{2\times 2}(\mathbb{R})$ and basis $B_2 = \{1\}$ of $\mathbb{R}$.

2. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x_1, x_2, x_3) = (x_1 - x_3, x_1, 2x_1 + x_2)$.

(a) Write the matrix representation A of T with respect to standard basis of $\mathbb{R}^3$.

(b) Obtain the matrix representation B of T with respect to ordered basis $\{(1, 1, 0), (0, 1, 1), (2, 2, 3)\}$ of $\mathbb{R}^3$ by using the formula $B = S^{-1}AS$ where S is the matrix whose columns are given by change of basis.

3. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation such that the matrix representation of T with respect to bases $\{(1, 2), (0, 1)\}$ of $\mathbb{R}^2$ and $\{(1, 1, 0), (0, 1, 1), (2, 2, 3)\}$ of $\mathbb{R}^3$ be $\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$. Find $T(x, y)$.