Homework 6

1. Prove that there does not exist a linear transformation from $\mathbb{R}^5$ to $\mathbb{R}^2$ whose null space equals
$$\{(x_1, x_2, x_3, x_4, x_5) \in \mathbb{R}^5 : x_1 = 3x_2 \text{ and } x_3 = x_4 = x_5\}.$$

2. Let V be a vector space and $T : V \to V$ be a linear transformation such that $T(T(v)) = T(v)$ for each $v \in V$. Prove that
$$V = null(T) \oplus Im(T)$$