Homework 2

1. Give an example of a nonempty subset U of $\mathbb{R}^2$ such that U is closed under addition (meaning for each $a, b \in U$, $a+b \in U$) and under taking inverses (meaning $-u \in U$ whenever $u \in U$), but U is not a subspace of $\mathbb{R}^2$.

2. Give an example of a nonempty subset U of $\mathbb{R}^2$ such that U is closed under scalar multiplication (meaning for each $a \in U$, for each $\alpha \in \mathbb{R}$, $\alpha a \in U$), but U is not a subspace of $\mathbb{R}^2$.

3. Let V be a vector space. Prove that if $\{v_1, v_2, \ldots, v_n\}$ is a linearly independent subset of V, then $\{v_1 - v_2, v_2 - v_3, \ldots, v_{n-1} - v_n, v_n\}$ is also linearly independent.

4. Suppose $\{v_1, v_2, \ldots, v_n\}$ is a linearly independent subset of V and $w \in V$. Prove that if $\{v_1+w, v_2+w, \ldots, v_n+w\}$ is linearly dependent, then $w \in span(v_1, v_2, \ldots, v_n)$.