Homework 1

1. Let $A = \{(x_1, x_2) : x_1, x_2 \in \mathbb{R}\}$. Define addition and scalar multiplication on A as follows:

$(x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2)$, and
$\alpha(x_1, x_2) = (\alpha x_1, x_2)$ for $\alpha \in \mathbb{R}$.

Is A, with the above defined addition and scalar multiplication, a vector space over $\mathbb{R}$?

2. Let $\mathbb{R}^3 = \{(x_1, x_2, x_3) : x_1, x_2, x_3 \in \mathbb{R}\}$. Define addition and scalar multiplication on $\mathbb{R}^3$ in the standard way as follows:

$(x_1, x_2, x_3) + (y_1, y_2, y_3) = (x_1 + y_1, x_2 + y_2, x_3 + y_3)$, and
$\alpha(x_1, x_2, x_3) = (\alpha x_1, \alpha x_2, \alpha x_3)$ for $\alpha \in \mathbb{R}$.

Is $\mathbb{R}^3$, with the above defined addition and scalar multiplication, a vector space over $\mathbb{R}$?

3. For each of the following subsets of $\mathbb{R}^3$, determine whether it is a subspace of $\mathbb{R}^3$:

(a) $U = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 + 2x_2 + 3x_3 = 0\}$

(b) $W = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 x_2 x_3 = 0\}$

(c) $V = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 + x_2 + x_3 = 1\}$