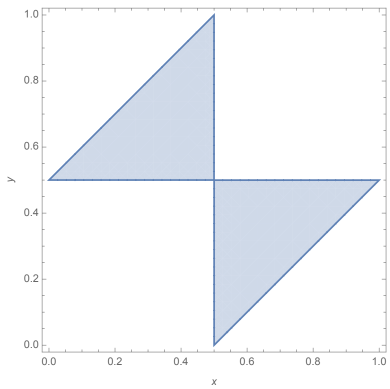
Billiken Challenge Solutions
January 2019

Triangle Making

Randomly select two points on a line segment, and cut the segment at those points. What is the probability that the three resulting segments can be formed into a triangle?

Solution: The probability is $1/4$. Assume the segment has length one. Let $x \in [0, 1]$ and $y \in [0, 1]$ be the two points. For three segments to form a triangle, the sum of lengths of any two of the segments must exceed the length of the other segment. Since the total segment length is one, this happens when each segment has length less than $1/2$. If $x < y$, the segments have lengths x, $y - x$, and $1 - y$. We need three inequalities to hold: $x < 1/2$, $y > 1/2$, $y - x < 1/2$. Similarly, if $x > y$ we require $y < 1/2$, $x > 1/2$, $x - y < 1/2$. These inequalities hold in the region shown below, which has area $1/4$.



This problem was suggested by Professor Greg Marks.

Triangle Placing

Given an infinite checkerboard and an arbitrary triangle. Is it possible to place the triangle on the board so that all three of its corners lie inside dark squares?

Solution: Put one vertex A of the triangle on a horizontal edge between a dark and light square. Now rotate the triangle around A until vertex B lies on a vertical edge. Slide A horizontally and B vertically along their respective edges. This will move C along a path which is not a horizontal or vertical line, so at some point C will be on the interior of a square while neither A nor B will be on a corner.

Suppose C is inside a dark square. A is on the horizontal edge of a dark square, so a small vertical translation will put A into the dark square while leaving C still inside a dark square and B still on a vertical edge between dark and light squares. Now a horizontal translation will put B into a dark square while leaving A and C still in the interior of dark squares. So, all vertices are inside dark squares as desired.

If C was originally in a light square, the same argument shows the triangle can be placed with all vertices inside light squares, and then translating up by one unit puts the vertices into dark squares.