1. Chapter 2.1 Logical Form and Logical Equivalence

1.1. Deductive Logic.

- An Argument is a sequence of statements aimed at demonstrating the truth of an assertion.
- The assertion at the end of the sequence is called the Conclusion, and the preceding statements are called Premises.
- To illustrate the logical form of arguments, we use letters of the alphabet (such as $p$, $q$, and $r$) to represent the component sentences of an argument.

1.2. Statements and Truth Tables.

- A Statement (or Proposition) is a sentence that is true or false but not both. For example: Two plus two equals four. For example: Two plus two equals five. For example: $x + y > 0$.
- If sentences are to be statements, they must have well-defined Truth Values—they must either be true or false. We can use a truth table to summarize truth values.
- If $p$ is a statement variable, the negation of $p$ is "not $p" or "It is not the case that $p" and is denoted $\sim p$. It has opposite truth value from $p$; if $p$ is true, $\sim p$ is false; if $p$ is false, $\sim p$ is true.

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<th>$p$</th>
<th>$\sim p$</th>
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- If $p$ and $q$ are statement variables, the conjunction of $p$ and $q$ is ",p and $q," denoted $p \land q$. It is true when, and only when, both $p$ and $q$ are true. If either $p$ or $q$ is false, or if both are false, $p \land q$ is false.

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- If $p$ and $q$ are statement variables, the Disjunction of $p$ and $q$ is ",p or $q," denoted $p \lor q$. It is true when either $p$ is true, or $q$ is true, or both $p$ and $q$ are true; it is
false only when both $p$ and $q$ are false.

- NOTE: The use of "or" in mathematics refers to the inclusive sense of the word. If you want to use the exclusive meaning you need to express "$p$ or $q$ but not both".

\[
\begin{array}{ccc}
 p & q & p \lor q \\
 T & T & T \\
 T & F & T \\
 F & T & T \\
 F & F & F \\
\end{array}
\]

For example: Write down the truth table for $(p \lor q) \land \sim (p \land q)$.

\[
\begin{array}{cccccc}
 p & q & p \lor q & p \land q & \sim (p \land q) & (p \lor q) \land \sim (p \land q) \\
 T & T & T & T & F & F \\
 T & F & T & F & T & F \\
 F & T & F & F & T & T \\
 F & F & F & F & T & F \\
\end{array}
\]

1.3. **In Class Group Work.** Section 2.1, Page 37: Answer all of question 6, 8 parts a and d, 16, and 18.
2. Logical Equivalence

- Two *statement forms* are called **logically equivalent** if, and only if, they have identical truth values for each possible substitution of statements for their statement variables. The logical equivalence of statement forms $P$ and $Q$ is denoted by writing $P \equiv Q$.

- Two *statements* are called **logically equivalent** if, and only if, they have logically equivalent forms when identical component statement variables are used to replace identical component statements.

  For example: $\sim (\sim p) \equiv p$

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For example: $\sim (p \land q)$ is not logically equivalent to $\sim p \lor \sim q$

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<th>$p \land q$</th>
<th>$\sim (p \land q)$</th>
<th>$\sim p \lor \sim q$</th>
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2.1. In Class Group Work. Is $\sim (p \land q)$ logically equivalent to $\sim p \lor \sim q$?
• The negation of an and statement is logically equivalent to the or statement in which each component is negated.
• The negation of an or statement is logically equivalent to the and statement in which each component is negated.
• These are called De Morgan’s Laws and they are MUY IMPORTANTE!

2.2. Tautologies and Contradictions.
• A Tautology is a statement form that is always true regardless of the truth values of the individual statements substituted for its statement variables. A statement whose form is a tautology is a tautological statement. (i.e., you always write a "T" for a tautology in your truth table; a tautology will produce ”all T’s”)
• A Contradiction is a statement form that is always false regardless of the truth values of the individual statements substituted for its statement variables. A statement whose form is a contradiction is a contradictory statement. (i.e., you always write a "F" for a contradiction in your truth table; a contradiction will produce "all F’s")
• Check out Theorem 2.11 on page 35 in section 2.1.

2.3. In Class Group Work. Use a truth table to show that $p \lor \sim p$ is a tautology and that $p \land \sim p$ is a contradiction.