

ABSTRACT ALGEBRA WITH GAP

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NOTE TO INSTRUCTOR

This note to instructors is intended to provide an overview of how the authors view the possible uses of this manual. Using software in an upper division mathematics course can have its place if it somehow provides a way for the student to better understand the material. There are at least five ways **GAP** can be a useful pedagogical tool:

- 1) as a fancy calculator,
- 2) as a way to provide large or complicated examples,
- 3) as a way for students to write simple computer algorithms,
- 4) as a way for producing large amounts of data so that the student can formulate a conjecture and
- 5) as a means for students to work in collaboration.

GAP can be used as a fancy calculator and thus eliminate some repetitive hand calculations that the student may have otherwise had to do. For example, once a student knows how to find all the conjugacy classes of a group of small order, **GAP** can be used to provide all the conjugacy classes of a group of order 80.

GAP has many built in functions, operations, and algebraic structures. Thus **GAP** can be used to quickly provide numerous examples, many more and of more complexity than could be done by hand. For example, all fourteen groups (and their properties) of order 16 could be easily examined during a small portion of a single class period, using **GAP**.

The students can begin writing simple computer algorithms using **GAP** commands. Writing an algorithm causes the student to solidify a new concept. For example, in order to write an algorithm that finds all the nilpotent elements in a group, the student has to be able to very precisely write code for checking nilpotency.

GAP also provides a means for producing large amounts of data quickly. A student can then look for patterns and formulate conjectures. When a student “discovers” a theorem before it is proved he may be more likely to remember and understand it. Also theorems that may be beyond the scope of the course could be introduced in this way. In some cases the patterns that develop help students understand the proof of the theorem or provide them with a possible approach to try to prove the theorem. In other cases the patterns that develop can be explained geometrically, and thus provide students with another way to understand the concept. Also in some cases, deceptive patterns can lead to incorrect conjectures and thus initiate an informative discussion on how to reformulate and test a new conjecture.

Doing projects on a computer lends itself to group work. For example, if a large amount of data is being produced in order to formulate a conjecture about a group of prime power order, students in

a group could each pick a certain number of primes to test and then compare their results as a group.

All five of the above pedagogical approaches are used in the exercises in this manual. The third, however, is not emphasized as much as the other four. The intent of this manual is to provide a supplement to a more traditional way of teaching abstract algebra. A course where the the main focus is to use abstract algebra concepts, learn how to prove abstract algebra theorems and understand abstract algebra structures, is assumed. We want to keep to a minimum the amount of time a student spends learning software code. Only an extremely small portion of the power of GAP is introduced to the students. In fact, some of the built in functions are purposely not introduced, as we do not want GAP to do too much of the work for the student. In addition, only the GAP commands that are going to be specifically used in a particular chapter are introduced.

The exercises in the later chapters of this lab manual do not assume that the student has worked all previous lab manual exercises. Thus exercises can be picked or skipped as the instructor views appropriate. On the other hand, the GAP material in Chapters -1 to Chapter 22 assumes that the GAP commands introduced in the text of the previous chapters has been covered. Thus we recommend that the student read each chapter of the lab manual as the corresponding material from the text is covered, even if no computer exercises are assigned. After Chapter 22, the GAP content no longer builds from chapter to chapter, and so the lab manual chapters can be done in any order from Chapters 23 - 33. We suggest instructors use the *Instructor Solution Manual* that accompanies this lab manual. It contains additional GAP commands and context which instructors will find useful when teaching with this software.