Publication List
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Papers are available online at http://math.binghamton.edu/sabalka .

(11) **On restricting subsets of bases in relatively free groups**, with Dmytro Savchuk. Accepted, *International Journal of Algebra and Computation*.

Let G be a free, free abelian, free nilpotent, or free solvable group, and let $A = \{a_1, \ldots, a_n\}$ be a basis for G. We prove that in many cases, if S is a subset of a basis for G which may be expressed as a word in A without using elements from $\{a_l, a_{l+1}, \ldots, a_n\}$, then S is a subset of a basis for the relatively free group on $\{a_1, \ldots, a_{l-1}\}$.

(10) **On the geometry of a proposed curve complex analogue for $Out(F_n)$**, with Dmytro Savchuk. Submitted.

The group $Out(F_n)$ of outer automorphisms of the free group has been an object of active study for many years, yet its geometry is not well understood. Recently, effort has been focused on finding a hyperbolic complex on which $Out(F_n)$ acts, in analogy with the curve complex for the mapping class group. Here, we focus on one of these proposed analogues: the edge splitting complex $\mathcal{ESC}$, equivalently known as the separating sphere complex. We characterize geodesic paths in its 1-skeleton $\mathcal{ES}$ algebraically, and use our characterization to find lower bounds on distances between points in this graph.

Our distance calculations allow us to find quasiflats of arbitrary dimension in $\mathcal{ESC}$. This shows that $\mathcal{ESC}$: is not hyperbolic, has infinite asymptotic dimension, and is such that every asymptotic cone is infinite dimensional. These quasiflats contain an unbounded orbit of a reducible element of $Out(F_n)$. As a consequence, there is no coarsely $Out(F_n)$-equivariant quasiisometry between $\mathcal{ESC}$ and other proposed curve complex analogues, including the regular free splitting complex, the (nontrivial intersection) free factorization complex, and the free factor complex, leaving hope that some of these complexes are hyperbolic.

(9) **Face vectors of subdivided simplicial complexes**, with Emanuele Delucchi and Aaron Pixton. Published online, *Discrete Mathematics* (2011). To appear in print.

In a recent paper of Brenti Welker, it was shown that for any simplicial n-dimensional complex X, the f-vectors of successive barycentric subdivisions of X have roots which converge to fixed values depending only on the dimension of X. We improve and generalize this result here. We begin with an alternative proof based on geometric intuition. We then observe and prove an interesting symmetry of these roots about the real number -2. This symmetry can be seen via a nice algebraic realization of barycentric subdivision as a simple map on formal power series in two variables. Finally, we use this algebraic machinery with some geometric motivation to generalize the combinatorial statements to arbitrary subdivision methods: any subdivision method will exhibit similar limit behavior and symmetry. Our techniques allow us to compute explicit formulas for the values of the limit roots in the case of barycentric subdivision.

(8) **Projection-forcing multisets of weight changes**, with Joshua Brown Kramer. *Journal of Combinatorial Theory, Series A*, 117(8):1136-1142, 2010.

Let $\mathbb{F}$ be a finite field. A multiset S of integers is *projection-forcing* if for every linear function $\phi : \mathbb{F}^n \to \mathbb{F}^m$ whose multiset of weight changes is S, ϕ is a coordinate projection up to permutation and scaling of entries. The MacWilliams Extension Theorem from coding theory says that $S = \{0, 0, \ldots, 0\}$ is projection-forcing. We give a (super-polynomial) algorithm to determine whether or not a given S is projection-forcing. We also give a condition that can be checked in polynomial time that implies that S is projection-forcing. This result is a generalization of the MacWilliams Extension Theorem and work by the first author.

(7) **Multidimensional online motion planning for a spherical robot**, with Joshua Brown Kramer. *International Journal of Computational Geometry and Applications*, 20(6):653-684, 2010.

We consider three related problems of robot movement in arbitrary dimensions: coverage, search, and navigation. For each problem, a spherical robot is asked to accomplish a motion-related task in an unknown environment whose geometry is learned by the robot during navigation. The robot is assumed to have tactile and global positioning sensors. We view these problems from the perspective of (non-linear) competitiveness as defined by Gabriely and Rimon. We first show that in 3 dimensions and higher, there is no upper bound on competitiveness: every online algorithm can do arbitrarily badly compared to the optimal. We then modify the problems by assuming a fixed clearance parameter. We are able to give optimally competitive algorithms under this assumption. We show these modified problems have polynomial competitiveness in the optimal path length, of degree proportional to the dimension minus 1.

(6) **Presentations of graph braid groups**, with Daniel Farley. Published online, *Forum Mathematicum* (2010). To appear in print.

Let Γ be a graph. The (unlabeled) configuration space $\mathcal{UC}^n\Gamma$ of n points on Γ is the space of n-element subsets of Γ. The n-strand braid group of Γ, denoted $B_n\Gamma$, is the fundamental group of $\mathcal{UC}^n\Gamma$. This paper extends the methods and results of (2). Here we describe how to compute presentations for $B_n\Gamma$, where n is an arbitrary natural number and Γ is an arbitrary finite connected graph. Particular attention is paid to the case $n = 2$, and many examples are given.

(5) **On rigidity and the isomorphism problem for tree braid groups**. *Groups, Geometry, and Dynamics*, 3(3):469-523, 2009.

We solve the isomorphism problem for braid groups on trees with $n = 4$ or 5 strands. We do so in three main steps, each of which is interesting in its own right. First, we establish some tools and terminology for dealing with computations using the cohomology of tree braid groups, couching our discussion in the language of differential forms. Second, we show that, given a tree braid group B_nT on $n = 4$ or 5 strands, $H^*(B_nT)$ is an exterior face algebra. Finally, we prove that one may reconstruct the tree T from a tree braid group B_nT for $n = 4$ or 5. Among other corollaries, this third step shows that, when $n = 4$ or 5, tree braid groups B_nT and trees T (up to homeomorphism) are in bijective correspondence. That such a bijection exists is not true for higher dimensional spaces, and is an artifact of the 1-dimensionality of trees. We end by stating the results for right-angled Artin groups corresponding to the main theorems, some of which do not yet appear in the literature.

(4) **On the cohomology rings of tree braid groups**, with Daniel Farley. *Journal of Pure and Applied Algebra*, 212(1):53-71, 2007.

Let Γ be a finite connected graph. The (unlabelled) configuration space $\mathcal{UC}^n\Gamma$ of n points on Γ is the space of n-element subsets of Γ. The n-strand braid group of Γ, denoted $B_n\Gamma$, is the fundamental group of $\mathcal{UC}^n\Gamma$. We use the methods and results of (2) to get a partial description of the cohomology rings $H^*(B_nT)$, where T is a tree. Our results are then used to prove that B_nT is a right-angled Artin group if and only if T is linear or $n < 4$. This gives a large number of counterexamples to Ghrist's conjecture that braid groups of planar graphs are right-angled Artin groups.

(3) **Embeddings of right-angled Artin groups into graph braid groups** *Geometriae Dedicata*, 124:191-198, 2007.

We construct an embedding of any right-angled Artin group $G(\Delta)$ defined by a graph Δ into a graph braid group. The number of strands required for the braid group is equal to the chromatic number of Δ. This construction yields an example of a hyperbolic surface subgroup embedded in a two strand planar graph braid group.

(2) **Discrete Morse theory and graph braid groups**, with Daniel Farley. *Algebraic and Geometric Topolology*, 5:1075-1109, 2005.

If Γ is any finite graph, then the *unlabelled configuration space of n points on Γ*, denoted $\mathcal{UC}^n\Gamma$, is the space of n-element subsets of Γ. The *braid group of Γ on n strands* is the fundamental group of $\mathcal{UC}^n\Gamma$. We apply a discrete version of Morse theory to these $\mathcal{UC}^n\Gamma$, for any n and any Γ, and provide a clear description of the critical cells in every case. As a result, we can calculate a presentation for the braid group of any tree, for any number of strands. We also give a simple proofof a theorem due to Ghrist: the space $\mathcal{UC}^n\Gamma$ strong deformation retracts onto a CW complex of dimension at most k, where k is the number of vertices in Γ of degree at least 3 (and k is thus independent of n).

(1) **Geodesics in the braid group on three strands**. In *Group theory, statistics, and cryptography*, volume 360 of *Contemporary Mathematics*, pages 133-150, 2004.

This is derived from my undergraduate thesis, advised by Susan Hermiller and John Meakin.

We study the geodesic growth series of the braid group on three strands, $B_3 := \langle a, b | aba = bab \rangle$. We show that the set of geodesics of B_3 with respect to the generating set $S := \{a, b\}^{\pm 1}$ is a regular language, and we provide an explicit computation of the geodesic growth series with respect to this set of generators. In the process, we give a necessary and sufficient condition for a freely reduced word $w \in S^*$ to be geodesic in B_3 with respect to S. Also, we show that the translation length with respect to S of any element in B_3 is an integer.