MT-A244-04  Final Exam  Fall 2007

You may keep this page of questions. Turn in all of your work on the colored paper.

FOR THIS EXAM

COMPUTERS AND CALCULATORS ARE NOT ALLOWED.

(1) 18 Points. Match the following equations with their graphs on the graphics page: (For uniformity, mark one of a,b,c,d,e or f next to each of the capital roman numbers I,II,III,IV,V and VI on the colored graphics page and turn in the graphics page.)

(a) \(x^2 = y^2 + z^2\)  \quad (b) \(y = z^2\)  \quad (c) \(x^2 + z^2 = 9\)

(d) \(x^2 - y^2 + z^2 = 9\)  \quad (e) \(y = x^2 + z^2 + 3\)  \quad (f) \(x^2 + y^2 + z^2 = 9\)

(2) 30 Points. Let \(v = i - 3j + 2k\) and \(w = 2i - j - 4k\).

(a) Find \(3v - w\).
(b) Find \(||w|||\).
(c) Find \(v \cdot w\).
(d) Find \(w \times v\).
(e) Find \(\cos \theta\) where \(\theta\) is the angle between \(v\) and \(w\).
(f) Find the direction cosines for the vector \(v\).

(3) 15 Points. Given \(z = f(x, y) = \ln(5 + x^2y^4)\), find the following:

(a) \(\frac{\partial z}{\partial x}\)  \quad (b) \(\frac{\partial z}{\partial y}\)  \quad (c) \(\frac{\partial^2 z}{\partial y^2}\).

(4) 15 Points. For this problem, \(w = f(x, y, z) = x^5y^2 + z^3\).

(a) Find the total differential, \(dw\).
(b) Find the gradient, \(\nabla w\).
(c) Find the directional derivative, \(f_v(1, 3, -1)\) if the direction vector is \(v = 2i - 2j + k\).

(5) 16 Points. For the following finite system of masses in the \(xy\)-plane, find the mass, \(M\), the moment, \(M_x\), about the \(x\)-axis, the moment, \(M_y\), about the \(y\)-axis, the center of mass, \((\bar{x}, \bar{y})\), and the moment of inertia, \(I_0\), for the system about the origin.

<table>
<thead>
<tr>
<th>(x)</th>
<th>-2</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>(m)</td>
<td>3</td>
<td>7</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>
(6) 18 Points. If a particle has position vector \( \mathbf{r} = \sin(5t)\mathbf{i} + \cos(5t)\mathbf{j} + e^{-7t}\mathbf{k} \), find

(a) the velocity vector \( \mathbf{v} \),
(b) the acceleration vector \( \mathbf{a} \), and
(c) the speed \( ||\mathbf{v}|| \).

(7) 16 Points. Evaluate the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \) if \( \mathbf{F} = z\mathbf{i} + x^2\mathbf{j} - xy\mathbf{k} \) and \( C \) is the straight line segment from the point \((0, -1, 3)\) to the point \((2, 2, 1)\).

(8) 12 Points. Find a vector parametric description for the line of intersection of the planes \( x - 3y + 2z = 6 \) and \( 2x + y - 4z = 12 \).

(9) 16 Points. Are the following statements true or false?

(a) If \( \mathbf{u} \) and \( \mathbf{v} \) are vectors, then \( \mathbf{u} \cdot \mathbf{v} \) is a vector.
(b) If \( f(t) \) is a function of a single variable, \( t \), and \( g(s, t) \) is a function of two variables, \( s \) and \( t \), then \( \frac{\partial}{\partial s} \left( f(t)g(s, t) \right) = f(t)g_s(s, t) \).
(c) The region consisting of all points \((x, y)\) satisfying \( x^2 + y^2 \geq 4 \) is bounded.
(d) If the point \( P_0 \) is a critical point for the function \( f \), then \( P_0 \) is either a local minimum or a local maximum for \( f \).
(e) If \( R \) is the region in the \( xy \)-plane that is inside the circle of radius 1 centered at the origin, then \( \iint_R \sin^2(xy) \, dA \leq 4 \).
(f) The iterated integrals \( \int_0^1 \int_0^2 f(x, y) \, dy \, dx \) and \( \int_0^1 \int_0^y f(x, y) \, dx \, dy \) always give the same value for any integrand \( f(x, y) \).
(g) The parametric curve \( \mathbf{r}(t) = (3t)\mathbf{i} + (t - 2)\mathbf{j} + (t^3)\mathbf{k} \) for \(-3 \leq t \leq 3\) passes through the origin.
(h) If a particle is moving along a parametrized curve \( \mathbf{r}(t) \), then the acceleration vector at any point is always perpendicular to the velocity vector at that point.

(10) 20 Points. Find and classify all of the critical points for the surface \( z = f(x, y) = -x^3 + 6xy + y^3 \).

(11) 24 Points. Find the moment of inertia about the \( z \)-axis for the solid region that is inside the sphere \( x^2 + y^2 + z^2 = 4 \), but outside the sphere \( x^2 + y^2 + z^2 = 1 \) if the solid has density \( \delta = \frac{1}{x^2 + y^2 + z^2} \) at a point \((x, y, z)\) within the solid.