Math 5220–Complex Analysis: Problem Set 5

Gill

Due: Friday February 23

Problem 1

Let $z \in \mathbb{C} \setminus \{0\}$ and $w \in \mathbb{C}$. Then the set z^w has one of four possible forms:

1. n distinct poits on a circle, where n is a positive integer.
2. Infinitely many points on a circle.
3. Infinitely many points which lie on infinitely many different rays through the origin, so that the points spiral around the origin at 0 and ∞.
4. !*#

(a) What is !*#?
(b) Take $z = i$. Give examples of w for which (1)-(4) occur, with $n = 3$ in (1), and sketch the sets i^w.

Problem 2

Let $f(z) = \exp(e^z)$. Find $\Re f(z), \Im f(z), |f(z)|$, and all solutions in $\mathbb{C}$ of $f(z) = 1$. Sketch this solution set in the plane. It is a two parameter family. Your final description may contain real terms of the form $\log x$ for $x > 0$, but may not use terms of the form $\log z$ where z is not a positive real.

Problem 3

Let $f(z) = \tan z$. Define $f(z) = \infty$ at points where $\cos z = 0$.
(a) Find a formula for the set $f^{-1}(w) = \arctan w$ in terms of sets log.
(b) $f(\mathbb{C}) = \mathbb{C}^* \setminus \{a, b\}$. What are the omitted values a and b?
(c) Describe and sketch the set $\arctan e^{i\theta}$ when $0 < \theta < \pi/2$. The rules of description are the same as in Problem 2. For maximal conciseness, the answer can contain $\log(\cot(\frac{\theta}{2} + \frac{\pi}{4}))$ or $\log(\tan(\frac{\pi}{4} - \frac{\theta}{2}))$.

Problem 4

Let $a \in \mathbb{C}, R > 0$, and P denote a polynomial. Find a formula for $\int_{|z-a|=R} P(z)d\overline{z}$ in terms of a, R, and the values of P and or its derivatives. Make the formula as simple as you can. To start, let $a = 0$.