Correct proofs.

**Theorem 1.** \( \forall n \in \mathbb{N}, \) 
\[
\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}
\]

**Proof.** \( \sum_{i=1}^{1} i^2 = 1 \) \( = \frac{1(1+1)(2(1)+1)}{6} \). Thus, \( \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \) for \( n = 1 \).

Let \( k \in \mathbb{N} \), and assume that \( \sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6} \).
We will show that, \( \sum_{i=1}^{k+1} i^2 = \frac{(k+1)(k+2)(2(k+1)+1)}{6} \)

\[
\sum_{i=1}^{k+1} i^2 = \sum_{i=1}^{k} i^2 + (k+1)^2
\]
\[
= \sum_{i=1}^{k} i^2 + (k+1)^2
\]
\[
= \frac{(k+1)k(2k+1)}{6} + (k+1)^2 \quad \text{by substituting the induction hypothesis}
\]
\[
= \frac{(k+1)k(2k+1) + 6(k+1)^2}{6}
\]
\[
= \frac{(k+1)[k(2k+1) + 6(k+1)]}{6}
\]
\[
= \frac{(k+1)(2k^2 + k + 6k + 6)}{6}
\]
\[
= \frac{(k+1)(2k^2 + 7k + 6)}{6}
\]
\[
= \frac{(k+1)(k+2)(2k+3)}{6}
\]
\[
= \frac{(k+1)(k+2)(2(k+1)+1)}{6}
\]

Thus, \( \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \) for all \( n \in \mathbb{N} \) by induction. \( \square \)
Theorem 2. For every non-negative integer $n$,

$$3|(2^{2n} - 1)$$

Proof. $2^{2(0)} - 1 = 0$. Thus, $3|(2^{2n} - 1)$ for $n = 0$.

Let $k \in \mathbb{Z}$ such that $k \geq 0$ and assume that $3|(2^{2k} - 1)$.
Thus, $2^{2k} - 1 = 3z$ for some $z \in \mathbb{Z}$.
We need to show that $3|(2^{2(k+1)} - 1)$.

$$2^{2(k+1)} - 1 = 2^{2k+2} - 1$$
$$= 4(2^{2k}) - 1$$
$$= 4(2^{2k} - 1 + 1) - 1$$
$$= 4(3z + 1) - 1$$
$$= 12z + 3$$
$$= 3(4z + 1)$$

Thus, $3|(2^{2(k+1)} - 1)$. This proves that $3|(2^{2n} - 1)$ for all $n \in \mathbb{N}$ by induction.

□