Concepts you need to know for workout problems on Test 2.

1. Definition of derivative as a limit.
2. Concept of derivative as instantaneous rate of change.
3. Concept of average rate of change and relationship to instantaneous rate of change.
4. Rates of change in various real world situations (what we covered Thurs. Oct 13)
5. Concept of derivative as the slope of the tangent line
6. Concept of secant line and relationship to the tangent line

7. Concept of how \( f'(a) \) and \( f'(a) \) can be used to find \( f'(b) \) if \( |b-a| \) is close to 0.

8. Implicit differentiation

Trig things you need to know

1. Pythagorean Theorem
   \[ a^2 + b^2 = c^2 \]
   \[ \sin^2(\theta) + \cos^2(\theta) = 1 \]

2. \( \sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \)
   \( \cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} \)
   \( \tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} \)

3. \( \frac{d}{dx} \sin(\theta), \frac{d}{dx} \cos(\theta), \frac{d}{dx} \tan^{-1}(x) \)
   \( \frac{d}{dx} \tan(\theta), \frac{d}{dx} \tan^{-1}(x) \)

4. \( \cos(\tan^{-1}(x)) \) - be able to figure out things like this.
3.9 Related Rates

Ex: A 10 ft ladder rests against a wall. If the bottom of the ladder slides away from the wall at a rate of 2 ft/s, \( \frac{dx}{dt} = 2 \), how fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?

Velocity is derivative of position with respect to time.

We want to find \( \frac{dy}{dt} \) when \( x = 6 \).

\[ x^2 + y^2 = 100 \]

\[ \frac{d}{dt} (x^2 + y^2) = \frac{d}{dt} (100) \]

\[ 2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0 \]

\[ \frac{dx}{dt} = 2, \quad x = 6 \]

Substitute only at the very end.

\[ x^2 + y^2 = 100 \]

\[ 6^2 + y^2 = 100 \]

\[ y^2 = 64 \]

\[ y = 8 \]

\[ 2 \cdot 6 \cdot 2 + 2 \cdot 8 \cdot \frac{dy}{dt} = 0 \]

\[ 24 + 16 \frac{dy}{dt} = 0 \]

\[ \frac{dy}{dt} = -\frac{24}{16} = -\frac{3}{2} \text{ ft/second} \]
Ex: A prisoner is escaping from jail and is running 10 ft/s along the prison wall. A spotlight 8 ft away from the wall is tracking the prisoner as he moves. How quickly is the spotlight rotating when the prisoner is 6 ft away from the closest point on the wall to the spotlight?

We want to know: \[ \frac{d\theta}{dt} \text{ when } x = 6 \]

\[ \tan(\theta) = \frac{x}{8} \]

\[ \frac{d}{dt} \left( \tan(\theta) \right) = \frac{dx}{dt} \left( \frac{x}{8} \right) = \frac{dx}{dt} \cdot \left( \frac{1}{8} x \right) \]

\[ \sec^2(\theta) \frac{d\theta}{dt} = \frac{1}{8} \frac{dx}{dt} \]

\[ \sec^2(\theta) = \frac{1}{\cos^2(\theta)} \]

\[ \frac{d\theta}{dt} = \frac{1}{8} \frac{dx}{dt} \cos^2(\theta) \]

\[ x = 6 \]

\[ 8^2 + 6^2 = 10^2 \]

\[ \cos(\theta) = \frac{5}{10} = \frac{4}{5} \]

\[ \frac{dx}{dt} = 10 \]

\[ \frac{d\theta}{dt} = \frac{1}{8} \left( 10 \right) \left( \frac{4}{5} \right)^2 = \frac{4}{5} \text{ ft/second} \]
Ex: A coke can has a radius of its base equal to 2 cm. If there is a hole in the bottom of the can and coke is leaking out at $4 \text{ cm}^3/\text{s}$ then how quickly is the height of the liquid in the can decreasing?

Volume = area of base $\cdot$ height

$V = \pi r^2 \cdot h$

We want $\frac{dh}{dt}$

$$\frac{d}{dt} (V) = \frac{d}{dt} (\pi r^2 \cdot h)$$

$$\frac{dV}{dt} = \pi r^2 \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{\pi r^2} \cdot \frac{dV}{dt}$$

$$\frac{dV}{dt} = 4 \text{ cm}^3/\text{s}$$

$$\frac{dh}{dt} = \frac{1}{4 \pi} \text{ cm}^3/\text{s}$$