Math 2660 Homework 3 due Friday Sep 23

(1) Prove: \( \forall x \in \mathbb{Z}, \text{ if } 4x - x^2 \text{ is even then } x \text{ is even.} \)

(2) Prove: \( \forall x \in \mathbb{Z}, x^2 - 3x \text{ is even.} \)

(3) Prove: \( \forall x, y \in \mathbb{Z}, x + y \text{ is even if and only if } x \text{ and } y \text{ have the same parity.} \)

(4) Prove: \( \forall x \in \mathbb{Z}, x^2 \text{ is odd if and only if } 5x - 7 \text{ is even.} \)

(5) State the division algorithm.

(6) Write the following integers \( x \) in their division algorithm decomposition \( x = mq + r. \)
   a. \( x = 77 \) \( m = 8 \)
   b. \( x = 13 \) \( m = 52 \)
   c. \( x = -70 \) \( m = 9 \)

(7) Prove: \( \forall x \in \mathbb{Z}, 3|5x \text{ if and only if } 3|x. \)