Math 266 Homework 2 due Friday Sep 16

1. Identify each of the following as a statement, open sentence, or neither.
   a. If $2 < 0$ then $0 = 1$.  
   b. $x > 3$ or $x \leq 3$. 
   c. The 10 billionth digit of $\pi$ is 4.

2. Write the following open sentences in the simplest way using set notation.
   a. $(x \in A) \lor (x \in B)$ 
   b. $(x \in A) \land (\sim x \in B)$ 
   c. $(x \in A) \Rightarrow (x \in B)$

3. Identify the truth value of each of the following statements. Explain why.
   a. If Wiggles ate some cat food for breakfast then Wiggles is a dog.
   b. If Wiggles was president of the United States then he would be the best president ever.

4. For each of the following open sentences, give the set of real numbers in interval notation where it is true.
   a. $x < 10 \land x > 6$ 
   b. $|x| > 3 \Rightarrow |x| > 5$

5. Put the following sentences in the form "If ... then ...".
   a. It is necessary to buy a ticket in order to win the lottery.
   b. You can go to the party only if you are invited.

6. Use a truth table to prove that a conditional statement is logically equivalent to its contra-positive.

7. Negate the following statements and simplify.
   a. $\forall x \in S, P(x) \Rightarrow Q(x)$.
   b. $\forall x \in S \exists y \in R, P(x, y) \lor Q(x, y)$.
   c. Everyone either likes Steve or knows someone who likes Steve.
   d. Somebody will pass every test.

8. Identify each of the following as true or false. Explain why.
   a. $\forall x \in \mathbb{R} \exists y \in \mathbb{R}, x = y^2$
   b. $\exists x \in \mathbb{R} \forall y \in \mathbb{R}, xy = y$
   c. $\forall x \in \mathbb{R}, x \neq 0 \Rightarrow \exists y \in \mathbb{R}, xy = 3$
   d. $\forall x \in \mathbb{R}, |x| < 0 \Rightarrow 2 < 1$. 