1. Chapter 1 review

1) a. Does $3 = \{3\}$? \[\text{NO}\]
b. Is $3 \in \{3\}$? \[\text{YES}\]
c. Is $3 \subseteq \{3\}$? \[\text{NO}\]
d. Does $\{3\} = \{3, 3, 3\}$? \[\text{YES}\]
e. Is $\{x \in \mathbb{Z} | x > 0\} \subseteq \{x \in \mathbb{R} | x > 0\}$? \[\text{YES}\]

2) a. When does $(a, b) = (c, d)$? \[\text{when } a = c \text{ and } b = d\]
b. If $A = \{1, 2\}$ and $B = \{x, y, c\}$ then what is $A \times B$, $B \times A$, and $A \times A$?

$A \times B = \{(1, x), (1, y), (1, c), (2, x), (2, y), (2, c)\}$

$B \times A = \{(x, 1), (y, 1), (c, 1), (x, 2), (y, 2), (c, 2)\}$

$A \times A = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

3) Let $A = \{2, 3, 5\}$ and $B = \{3, 4, 7\}$. Define a relation $T$ from $A$ to $B$ by: If $(x, y) \in A \times B$ then $(x, y) \in T$ means that $\frac{x}{y} \in \mathbb{Z}$.

a. Is $2 \in T$? Is $(2, 4) \in T$? \[\text{NO}, \text{NO}\]
b. Is $T : A \rightarrow B$ a function? why or why not?

\[\text{No, there is no } y \in B \text{ so that } (2, y) \in T\]

c. Write down $T$ as a set of ordered pairs.

$T = \{(3, 3)\}$

d. Draw an arrow diagram for $T$. 

![Diagram](image-url)
2. Chapter 2 Review

1) Construct a truth table for \((p \land q) \lor (\sim p \land \sim q)\).

<table>
<thead>
<tr>
<th>(p)</th>
<th>(q)</th>
<th>(\sim p)</th>
<th>(\sim q)</th>
<th>(p \land \sim q)</th>
<th>(\sim p \land \sim q)</th>
<th>((p \land q) \lor (\sim p \land \sim q))</th>
</tr>
</thead>
<tbody>
<tr>
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2) Construct a truth table to show that \(\sim (p \lor q)\) is logically equivalent to \(\sim p \land \sim q\).

What is the name for this law? **Demorgan's Law**

<table>
<thead>
<tr>
<th>(p)</th>
<th>(q)</th>
<th>(\sim p)</th>
<th>(\sim q)</th>
<th>(p \lor \sim q)</th>
<th>(\sim p \land \sim q)</th>
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</table>

3) Construct a truth table to show that \(p \rightarrow q\) is not logically equivalent to \(\sim p \land \sim q\).

<table>
<thead>
<tr>
<th>(p)</th>
<th>(q)</th>
<th>(\sim p)</th>
<th>(\sim q)</th>
<th>(p \land \sim q)</th>
<th>(\sim p \land \sim q)</th>
<th>(p \rightarrow q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
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4) Use a truth table to determine if the following argument is logically valid. Write a sentence which justifies your conclusion.

\[ p \rightarrow q \lor \sim r \]
\[ q \rightarrow p \land r \]
\[ \therefore p \rightarrow r \]

The argument is not logically valid because there is a row in which both the premises are true but the conclusion is false.
You will be provided with the following information on the test.

2.1. Modus Ponens and Modus Tollens.

- The **modus ponens** argument form has the following form:
  
  If $p$ then $q$.
  
  $p$
  
  $\therefore q$.

- **Modus tollens** has the following form:
  
  If $p$ then $q$.
  
  $\sim q$
  
  $\therefore \sim p$.

2.2. Additional Valid Argument Forms: Rules of Inference.

- A rule of inference is a form of argument that is valid. Modus ponens and modus tollens are both rules of inference. Here are some more...

<table>
<thead>
<tr>
<th>Generalization</th>
<th>$p$</th>
<th>$\therefore p \lor q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specialization</td>
<td>$p \land q$</td>
<td>$\therefore p$</td>
</tr>
<tr>
<td>Proof by Division into Cases</td>
<td>$p \lor q$</td>
<td>$\therefore p$</td>
</tr>
<tr>
<td></td>
<td>$p \rightarrow q$</td>
<td>$\therefore q$</td>
</tr>
<tr>
<td></td>
<td>$q \rightarrow r$</td>
<td>$\therefore r$</td>
</tr>
<tr>
<td>Contradiction Rule</td>
<td>$\sim(p \rightarrow c)$</td>
<td>$\therefore p$</td>
</tr>
</tbody>
</table>

| Elimination | $p \lor q$ | $\sim q$ | $\therefore p$ |
| Transitivity | $p \rightarrow q$ | $q \rightarrow r$ | $\therefore p \rightarrow r$ |
| Conjunction | $p$ | $q$ | $\therefore p \land q$ |
5) Write a logical argument which determines what I ate for dinner. Number each step in your argument and cite which rule you use for each step.
   a. I did not have a coupon for hamburger buns.
   b. I had hamburgers or chicken for dinner.
   c. If I had hamburgers for dinner then I bought hamburger buns.
   d. If I did not have a coupon for hamburger buns then I did not buy hamburger buns.

1. I did not buy hamburger buns (by a and Modus Ponens)
2. I did not have hamburgers for dinner (by c and Modus Tollens)
3. I had chicken for dinner (by b and elimination)
3. Chapter 3 review

1) a. Give an example of a universal conditional statement.
\[ \forall x \in \mathbb{R}, \text{ if } x > 0 \text{ then } x^3 > 0 \]

b. Write the contrapositive of the example.
\[ \forall x \in \mathbb{R}, \text{ if } x^3 \neq 0 \text{ then } x \neq 0 \]

c. Write the negation of the example.
\[ \exists x \in \mathbb{R}, \text{ such that } x > 0 \text{ and } x^3 \neq 0 \]

2) Write the following statements symbolically using \( \forall, \exists, \vee, \wedge, \rightarrow \). Then write their negation.

a. If \( x, y \in \mathbb{R} \) then \( x + y \in \mathbb{R} \).
\[ \forall x, y \in \mathbb{R}, x + y \in \mathbb{R} \]

b. Every real number \( x \) has an additive inverse \( y \).
\[ \forall x \in \mathbb{R}, \exists y \in \mathbb{R} \text{ such that } x + y = 0. \]
\[ \exists x \in \mathbb{R}, \text{ such that } \forall y \in \mathbb{R} x + y \neq 0. \]

c. Being divisible by 8 is not a necessary condition for an integer to be divisible by 4.
\[ \exists x \in \mathbb{Z}, \text{ such that } (x \text{ is divisible by 4}) \wedge (x \text{ is not divisible by 8}) \]
\[ \forall x \in \mathbb{Z}, x \text{ is divisible by 4} \rightarrow x \text{ is divisible by 8} \]

d. If I am tired and at home then I will go to sleep.
\[ \forall t \text{ times } t \quad \text{I am tired at } t \text{ } \land \text{I am at home at } t \rightarrow \text{I will go to sleep at } t \]
\[ \exists t \text{ a time } t \text{ such that } (\text{I am tired at } t) \land (\text{I am at home at } t) \land (\text{I will not go to sleep at } t). \]
2) Is it true or false that every real number bigger than 4 and less than 3 must be negative? Explain why.

True, there is no number bigger than 4 and less than 3, thus the statement is vacuously true.

3) Use diagrams to show whether the following arguments are logically valid or invalid.

a. All people are mammals.
   All mammals are mortal.
   ∴ All people are mortal.

   ![Diagram showing valid reasoning]

   **Valid**

a. Discrete math problems are fun.
   This problem is fun.
   ∴ This problem is discrete math.

   ![Diagram showing invalid reasoning]

   **Invalid**