Theorem 1.1 (Generalized Pigeonhole Principle). For any function $f$ from a finite set $X$ with $n$ elements to a finite set $Y$ with $m$ elements and for any positive integer $k$, if $k < n/m$, then there is some $y \in Y$ such that $y$ is the image of at least $k + 1$ distinct elements of $X$.

Theorem 1.2 (Generalized Pigeonhole Principle (Contrapositive)). For any function $f$ from a finite set $X$ with $n$ elements to a finite set $Y$ with $m$ elements and for any positive integer $k$, if for each $y \in Y$, $f^{-1}(y)$ has at most $k$ elements, then $X$ has at most $km$ elements; in other words, $n \leq km$. 
(1) There are 42 students who share 12 computers. Each student uses exactly 1 computer, and no computer is used by more than 6 students. Show that at least 5 computers are used by 3 or more students.
2. Chapter 9.5: Counting Subsets of a Set: Combinations

Definition. Let \( n \) and \( r \) be nonnegative integers with \( r \leq n \). An \( r \)-combination of a set of \( n \)-elements is a subset of \( r \) of the \( n \) elements. The total number of \( r \)-combinations of a set of \( n \)-elements is denoted:

\[
\binom{n}{r}
\]

This notation is called \( n \) choose \( r \).

Example: What are all the 2 combinations of \( \{a, b, c, d\} \)?

Example: What are all the 3 combinations of \( \{a, b, c, d\} \)?

Theorem 2.1. For all \( n, r \in \mathbb{N} \) with \( r \leq n \):

\[
\binom{n}{r} = \frac{n!}{r!(n-r)!}
\]
In class work:

(1) How many ways are there to choose 6 people from a group of 10 to work as a team?

(2) How many ways are there to choose 6 people from a group of 10 to work as a team if there are 2 who insist on being together? That is the team must contain both of them or neither.

(3) How many ways are there to choose 6 people from a group of 10 to work as a team if there are 2 who insist on not working together? That is the team must not contain both of them.