1. Chapter 4 review

1) Prove using the definition of even: For all integers \( n \), if \( n \) is even then \( (-1)^n = 1 \).

2) Prove using the definition of odd: The product of any two odd integers is odd.
3) Prove: For each integer $n$ with $1 \leq n \leq 5$, $n^2 - n + 11$ is prime.

4) Show that: $.123123123...$ is a rational number.

5) Prove using the definition of divides: For all integers $a, b,$ and $c$, if $a$ divides $b$ and $b$ divides $c$ then $a$ divides $c$. 
6) Evaluate 50 \texttt{div} 4 and 50 \texttt{mod} 4.

7) Prove using the definition of \texttt{mod}: For every integer $p$, if $p \texttt{mod} 5 = 2$ then $4p \texttt{mod} 5 = 3$.

8) Prove: For all integers $n$, $n^2 - n$ is even.
9 a) How do you prove a statement by contradiction?

9 b) Prove: There is no greatest negative rational number.

9 a) How do you prove “∀x ∈ D, if P(x) then Q(x)” by contraposition?

9 b) Prove: For all integers a, b, and c, if a ∤ bc then a ∤ b.
2. Chapter 5 review

10) a. Compute: \( \sum_{i=2}^{5} i^2 \)

b. Compute: \( \prod_{i=-5}^{-1} i \)

c. Compute: \( \sum_{i=1}^{100} (i)^2 - (i - 1)^2 \)

d. Compute: \( \prod_{i=2}^{50} \frac{i(i-1)}{(i+1)(i+2)} \)

b. Compute $\binom{100}{99}$

c. Compute $\binom{10}{8}$

d. Expand $(x - 1)^5$. 
12 a) What are the steps for proving “For all integers \( n \) such that \( n \geq a \), \( P(n) \) is true” using mathematical induction?

12 b) Prove: For all integers \( n \geq 1 \),

\[
\sum_{i=1}^{n} i = \frac{n(n + 1)}{2}.
\]
13) Prove: For all integers \( n \geq 0 \),

\[
\sum_{i=1}^{n+1} i2^i = n2^{n+1} + 2
\]
14 a) What are the steps for proving “For all integers $n$ such that $n \geq a$, $P(n)$ is true” using strong mathematical induction?

14 b) Prove: If $s_0 = 0$, $s_1 = 4$, and $s_k = 6s_{k-1} - 5s_{k-2}$ $\forall k \geq 2$, then $s_n = 5^n - 1$ for all integers $n \geq 0$. 
3. **Chapter 6 review**

15) Let $B = \{n \in \mathbb{Z} \mid n = 21r + 10 \text{ for some } r \in \mathbb{Z}\}$ and $C = \{m \in \mathbb{Z} \mid m = 7s + 3 \text{ for some } s \in \mathbb{Z}\}$. Prove that $B \subseteq C$.

16) Let $A = \{a, b, c, d, e\}$, $B = \{d, e, f, g\}$ and $C = \{b, c, d, f\}$. What is $(A - B) \cap C$?
(1) Commutative Laws: For all sets $A$ and $B$,
\[ A \cup B = B \cup A \text{ and } A \cap B = B \cap A. \]

(2) Associative Laws: For all sets $A$, $B$, and $C$,
\[ (A \cup B) \cup C = A \cup (B \cup C) \text{ and } (A \cap B) \cap C = A \cap (B \cap C). \]

(3) Distributive Laws: For all sets $A$, $B$, and $C$,
\[ A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \text{ and } A \cap (B \cup C) = (A \cap B) \cup (A \cap C). \]

(4) Identity Laws: For all sets $A$,
\[ A \cup \emptyset = A \text{ and } A \cap U = A. \]

(5) Complement Laws: For all sets $A$,
\[ A \cup A^c = U \text{ and } A \cap A^c = \emptyset. \]

(6) Double Complement Law: For all sets $A$,
\[ (A^c)^c = A. \]

(7) Idempotent Laws: For all sets $A$,
\[ A \cup A = A \text{ and } A \cap A = A. \]

(8) Universal Bound Laws: For all sets $A$,
\[ A \cup U = U \text{ and } A \cap \emptyset = \emptyset. \]

(9) De Morgan's Laws: For all sets $A$ and $B$,
\[ (A \cup B)^c = A^c \cap B^c \text{ and } (A \cap B)^c = A^c \cup B^c. \]

(10) Absorption Laws: For all sets $A$ and $B$,
\[ A \cup (A \cap B) = A \text{ and } A \cap (A \cup B) = A. \]

(11) Complements of $U$ and $\emptyset$:
\[ U^c = \emptyset \text{ and } \emptyset^c = U \]
(12) Set Difference Law: For all sets $A$ and $B$,

$$A - B = A \cap B^c$$
17) Prove the first part of the Distributive Law
18) Prove the first part of De Morgan's Law.
19) Prove or give a counterexample to the statement: For all sets $A, B, \text{ and } C$, 

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$
20) Prove or give a counterexample to the statement: For all sets $A, B,$ and $C,$

$$A \cup (B - C) = (A \cup B) - (A \cup C)$$
21) Prove using the given set theory laws that for all sets $A$ and $B$,

$$A \cup (B - A) = A \cup B$$
22) Prove using the given set theory laws that for all sets $A$ and $B$,

$$(B^c \cup (B^c - A))^c = B$$