Definition (ordered n-tuple). Let \( n \) be a positive integer and let \( x_1, x_2, \ldots, x_n \) be \( n \) elements. \((x_1, x_2)\) is called an ordered pair, \((x_1, x_2, x_3)\) is called an ordered triple, and \((x_1, x_2, \ldots, x_n)\) is called an ordered n-tuple. Two n-tuples \((x_1, x_2, \ldots, x_n)\) and \((y_1, y_2, \ldots, y_n)\) are equal if and only if \( x_1 = y_1, x_2 = y_2, \ldots, x_n = y_n \). That is:

\[(x_1, x_2, \ldots, x_n) = (y_1, y_2, \ldots, y_n) \iff x_1 = y_1, x_2 = y_2, \ldots, x_n = y_n\]

Examples:
- Is \((1, 2, 3) = (1, 2, 3, 4)\)?
- Is \(\{1, 2, 3\} = (1, 2, 3)\)?
- Is \((1, 2, 3) = (1, 3, 2)\)?
Basic set relations:

(1) Inclusion of Intersections: For all sets $A$ and $B$,
$$A \cap B \subseteq A \text{ and } A \cap B \subseteq B.$$ 

(2) Inclusion in Union: For all sets $A$ and $B$,
$$A \subseteq A \cup B \text{ and } B \subseteq A \cup B.$$ 

(3) Transitive Property of Subsets: For all sets $A$, $B$, and $C$,
If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$. 

Procedural Versions of Set Definitions:

Let $X$ and $Y$ be subsets of a universal set $U$ and suppose $x, y \in U$.

(1) $x \in X \cup Y \iff x \in X$ or $x \in Y$

(2) $x \in X \cap Y \iff x \in X$ and $x \in Y$

(3) $x \in X - Y \iff x \in X$ and $x \notin Y$

(4) $x \in X^c \iff x \notin X$

(5) $(x, y) \in X \times Y \iff x \in X$ and $y \in Y$

Example: Prove the Inclusion of Intersections relation
Class work:
(1) Prove the Inclusion in Union relation

(2) Prove the Transitive Property of Subsets
(1) Commutative Laws: For all sets $A$ and $B$,
$$ A \cup B = B \cup A \quad \text{and} \quad A \cap B = B \cap A. $$

(2) Associative Laws: For all sets $A$, $B$, and $C$,
$$ (A \cup B) \cup C = A \cup (B \cup C) \quad \text{and} \quad (A \cap B) \cap C = A \cap (B \cap C). $$

(3) Distributive Laws: For all sets $A$, $B$, and $C$,
$$ A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \quad \text{and} \quad A \cap (B \cup C) = (A \cap B) \cup (A \cap C). $$

(4) Identity Laws: For all sets $A$,
$$ A \cup \emptyset = A \quad \text{and} \quad A \cap \emptyset = \emptyset. $$

(5) Complement Laws: For all sets $A$,
$$ A \cup A^c = U \quad \text{and} \quad A \cap A^c = \emptyset. $$

(6) Double Complement Law: For all sets $A$,
$$ (A^c)^c = A. $$

(7) Idempotent Laws: For all sets $A$,
$$ A \cup A = A \quad \text{and} \quad A \cap A = A. $$

(8) Universal Bound Laws: For all sets $A$,
$$ A \cup U = U \quad \text{and} \quad A \cap \emptyset = \emptyset. $$

(9) De Morgan’s Laws: For all sets $A$ and $B$,
$$ (A \cup B)^c = A^c \cup B^c \quad \text{and} \quad (A \cap B)^c = A^c \cup B^c. $$

(10) Absorption Laws: For all sets $A$ and $B$,
$$ A \cup (A \cap B) = A \cup A \quad \text{and} \quad A \cap (A \cup B) = A. $$

(11) Complements of $U$ and $\emptyset$:
$$ U^c = \emptyset \quad \text{and} \quad \emptyset^c = U $$

(12) Set Difference Law: For all sets $A$ and $B$,
$$ A - B = A \cap B^c $$
How to prove two sets are equal:
Let $X$ and $Y$ be two sets. The following steps prove that $X = Y$:

1. Prove that $X \subseteq Y$
2. Prove that $Y \subseteq X$

Examples:

1. Prove the second Commutative Law

2. Prove the first Distributive Law
In class work:
(1) Prove the first Complement Law

(2) Prove the Double Complement Law
(3) Prove the first Absorption Law

(4) Prove the Set Difference Law
(5) Prove De Morgan's Laws