1. Chapter 1 review

1) a. Does \(3 = \{3\}\)?
   b. Is \(3 \in \{3\}\)?
   c. Is \(3 \subseteq \{3\}\)?
   d. Does \(\{3\} = \{3, 3, 3, 3\}\)?
   e. Is \(\{x \in \mathbb{Z} | x > 0\} \subseteq \{x \in \mathbb{R} | x > 0\}\)?

2) a. When does \((a, b) = (c, d)\)?
   b. If \(A = \{1, 2\}\) and \(B = \{x, y, c\}\) then what is \(A \times B, B \times A,\) and \(A \times A\)?

3) Let \(A = \{2, 3, 5\}\) and \(B = \{3, 4, 7\}\). Define a relation \(T\) from \(A\) to \(B\) by: If \((x, y) \in A \times B\) then \((x, y) \in T\) means that \(\frac{x}{y} \in \mathbb{Z}\).
   a. Is \(2T7\)? Is \((2, 4) \in T\)?
   b. Is \(T : A \rightarrow B\) a function? why or why not?
   c. Write down \(T\) as a set of ordered pairs.
   d. Draw an arrow diagram for \(T\).
2. Chapter 2 review

1) Construct a truth table for \((p \land q) \lor (\sim p \land \sim q)\).

2) Construct a truth table to show that \(\sim (p \lor q)\) is logically equivalent to \(\sim p \land \sim q\). What is the name for this law?

3) Construct a truth table to show that \(p \rightarrow q\) is not logically equivalent to \(\sim p \land \sim q\).

4) Use a truth table to determine if the following argument is logically valid. Write a sentence which justifies your conclusion.

\[
\begin{align*}
p &\rightarrow q \lor \sim r \\
q &\rightarrow p \land r \\
\therefore p &\rightarrow r
\end{align*}
\]
You will be provided with the following information on the test.

2.1. Modus Ponens and Modus Tollens.
- The modus ponens argument form has the following form:
  
  If \( p \) then \( q \).
  
  \( p \)
  
  \( \therefore q. \)

- Modus tollens has the following form:
  
  If \( p \) then \( q \).
  
  \( \sim q \)
  
  \( \therefore \sim p. \)

2.2. Additional Valid Argument Forms: Rules of Inference.
- A rule of inference is a form of argument that is valid. Modus ponens and modus tollens are both rules of inference. Here are some more...

<table>
<thead>
<tr>
<th>Generalization</th>
<th>( p )</th>
<th>( \therefore p \lor q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specialization</td>
<td>( p \land q )</td>
<td>( \therefore p )</td>
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Proof by Division into Cases

<table>
<thead>
<tr>
<th>( p \lor q )</th>
<th>( p \rightarrow r )</th>
<th>( q \rightarrow r )</th>
<th>( \therefore r )</th>
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| Contradiction Rule | \( \sim p \rightarrow c \) | \( \therefore p \) |

| Elimination | \( p \lor q \) | \( \sim q \) | \( \therefore p \) |

Transitivity

| \( p \rightarrow q \) | \( q \rightarrow r \) | \( \therefore p \rightarrow r \) |

Conjunction

| \( p \) | \( q \) | \( \therefore p \land q \) |
5) Write a logical argument which determines what I ate for dinner. Number each step in your argument and cite which rule you use for each step.

a. I did not have a coupon for hamburger buns.
b. I had hamburgers or chicken for dinner.
c. If I had hamburgers for dinner then I bought hamburger buns.
d. If I did not have a coupon for hamburger buns then I did not buy hamburger buns.
6) Write a logical argument which determines how Blue Beard the pirate stashed his treasure. Number each step in your argument and cite which rule you use for each step.

a. Blue Beard buried his treasure on land or Blue Beard sunk his treasure in the lagoon.

b. The first mate helped Blue Beard stash the treasure or the cabin boy helped Blue Beard stash the treasure.

c. If the first mate helped Blue Beard then they stashed the treasure with some rum.

d. If the cabin boy helped Blue Beard then they used a treasure map.

e. If they sank the treasure in the lagoon then they did not use a treasure map.

f. If they did not stash the treasure with some rum then they did not bury the treasure.

g. If they stashed the treasure with some rum then they did not sink the treasure in the lagoon.
7) (Hard problem for fun) The prison warden gives three prisoners a chance to win their freedom. The warden puts a hat on each prisoners head and says that a prisoner will be allowed to go free if they can say what color hat they are wearing, but if they get it wrong then they will be executed. The first two prisoners can see, but the third prisoner is blind. Use the following information to determine what color hat each prisoner is wearing, assuming each prisoner is a perfect logician.

a. The warden owns 3 red hats and 2 white hats.
b. The warden asks the first prisoner what color hat he is wearing, but the first prisoner does not know.
c. The warden then asks the second prisoner what color hat he is wearing, but the second prisoner does not know.
d. The warden then asks the third prisoner what color hat he is wearing, and the third prisoner answers correctly.
3. Chapter 3 review

1) a. Give an example of a universal conditional statement.

   b. Write the contrapositive of the example.

   c. Write the negation of the example.

2) Write the following statements symbolically using \( \forall, \exists, \lor, \land, \rightarrow \). Then write their negation.
   a. If \( x, y \in \mathbb{R} \) then \( x + y \in \mathbb{R} \).

   b. Every real number \( x \) has an additive inverse \( y \).

   c. Being divisible by 8 is not a necessary condition for an integer to be divisible by 4.

   d. If I am tired and at home then I will go to sleep.
2) Is it true or false that every real number bigger than 4 and less than 3 must be negative? Explain why.

3) Use diagrams to show whether the following arguments are logically valid or invalid.
   a. All people are mammals.
      All mammals are mortal.
      ∴ All people are mortal.

   a. Discrete math problems are fun.
      This problem is fun.
      ∴ This problem is discrete math.