1. Chapter 9.1 Counting and Probability

The probability of an event occurring is a number between 0 and 1, and represents essentially how often that event occurs. For example:

- The probability of flipping a coin and it landing on heads is $\frac{1}{2}$.
- The probability of rolling a 6-sided die and getting the number 3 is $\frac{1}{6}$.

**Definition.** A sample space is the set of all possible outcomes of a random process. An event is a subset of a sample space. If $E$ is an event then we denote $N(E)$ to be the number of elements in $E$.

**Theorem 1.1** (Equally Likely Probability Formula). If $S$ is a finite sample space in which all outcomes are equally likely and $E$ is an event in $S$, then the probability of $E$, denoted $P(E)$, is the number of elements in $E$ divided by the number of elements in $S$. Symbolically this is:

$$P(E) = \frac{N(E)}{N(S)}$$

**Examples:**
- What is the probability of flipping two coins and getting both heads or both tails?
- What is the probability of flipping three coins and getting three heads or three tails?
The possible outcomes of rolling two six sided dice are as follows:

$Die1 : 1, 2, 3, 4, 5, 6, 1, 2, 3, 4, 5, 6, 1, 2, 3, 4, 5, 6, 1, 2, 3, 4, 5, 6$
$Die2 : 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2

There are 36 possible outcomes.

(1) What is the probability that the sum of the two numbers is 9?

(2) What is the probability that the sum of the two numbers is at most 5?

(3) What is the probability that at least one of the numbers is 1?

(4) What is the probability that both numbers are the same?
1.1. **Monty Hall Problem.** There are three doors on the set of a game show. Behind only one of the doors is a prize. We will call the doors A, B, and C. The game works as follows. You pick a door. Then the host picks a different door and shows you that it is empty. What is the probability that the door you picked has the prize? The host asks if you want to change your pick to a different door instead, should you do so?
2. 9.2 Possibility Trees and the Multiplication Rule

Teams A and B play each other repeatedly until one team wins 2 games in a row or a total of three games.

(1) How many ways can the tournament be played?

(2) What is the probability of 5 games being played?

(3) What is the probability of a team winning 2 games in a row?
A person is buying a computer from either Dell, HP, or Sony. The operating system will be either Windows or Linux. What are the total number of possible combinations?

**Theorem 2.1** (The Multiplication Rule). *If an operation consists of k steps, and the jth step can be performed in \( n_j \) ways for all \( 1 \leq j \leq k \), then the entire operation can be performed in \( n_1n_2\ldots n_k \) ways.*

(1) How many ways are there to roll 3 dice, each of which has 6 sides?

(2) How many ways are there to order a set of 5 elements?

(3) How many ways are there to draw 5 cards from a deck of 10 cards if the order in which you draw cards matters?

(4) How many ways are there to draw 5 cards from a deck of 10 cards if the order in which you draw cards does not matter?