1. Chapter 4 review

1) Prove using the definition of even: For all integers \( n \), if \( n \) is even then \((-1)^n = 1\).

2) Prove using the definition of odd: The product of any two odd integers is odd.
3) Prove: For each integer \( n \) with \( 1 \leq n \leq 5 \), \( n^2 - n + 11 \) is prime.

4) Show that: \( .123123123... \) is a rational number.

5) Prove using the definition of divides: For all integers \( a, b, \) and \( c \), if \( a \) divides \( b \) and \( b \) divides \( c \) then \( a \) divides \( c \).
6) Evaluate 50 \text{ div } 4 and 50 \text{ mod } 4.

7) Prove using the definition of \text{ mod}: For every integer \( p \), if \( p \text{ mod } 5 = 2 \) then \( 4p \text{ mod } 5 = 3 \).

8) Prove: For all integers \( n \), \( n^2 - n \) is even.
9 a) How do you prove a statement by contradiction?

9 b) Prove: There is no greatest negative rational number.

9 a) How do you prove “∀x ∈ D, if P(x) then Q(x)” by contraposition?

9 b) Prove: For all integers a, b, and c, if a ∤ bc then a ∤ b.
2. Chapter 5 review

1) a. Compute: \( \sum_{i=2}^{5} i^2 \)

b. Compute: \( \prod_{i=-5}^{-1} i \)

c. Compute: \( \sum_{i=1}^{100} (i)^2 - (i - 1)^2 \)

d. Compute: \( \prod_{i=2}^{50} \frac{i(i-1)}{(i+1)(i+2)} \)
2) a. Compute $5!$. 

b. Compute $\binom{100}{99}$ 

c. Compute $\binom{10}{8}$ 

d. Expand $(x - 1)^5$. 
3 a) What are the steps for proving “For all integers \( n \) such that \( n \geq a \), \( P(n) \) is true” using mathematical induction?

3 b) Prove: For all integers \( n \geq 1 \),

\[
\sum_{i=1}^{n} i = \frac{n(n + 1)}{2}.
\]
4) Prove: For all integers \( n \geq 0 \),
\[
\sum_{i=1}^{n+1} i2^i = n2^{n+1} + 2
\]
6 a) What are the steps for proving “For all integers \( n \) such that \( n \geq a \), \( P(n) \) is true” using strong mathematical induction?

6 b) Prove: If \( s_0 = 0 \), \( s_1 = 4 \), and \( s_k = 6s_{k-1} - 5s_{k-2} \) \( \forall k \geq 2 \), then \( s_n = 5^n - 1 \) for all integers \( n \geq 0 \).