1. Chapter 5.2 and 5.3 Mathematical Induction I and II

Proving a statement by mathematical induction is a two step process the first step is called the basis step, the second step is called the inductive step.

**Principle of Mathematical Induction**
Consider a statement of the form, "For all integers $n \geq a$, a property $P(n)$ is true." To prove such a statement, perform the following two steps:

1. (basis step) Prove that $P(a)$ is true.
2. (inductive step) Let $k \geq a$ and assume $P(k)$ is true. Prove that $P(k + 1)$ is true.

This proves that for all integers $n \geq a$, $P(n)$ is true.

Induction can be illustrated by the following chain of implications:

$$P(a) \text{ is true } \Rightarrow P(a + 1) \text{ is true } \Rightarrow P(a + 2) \text{ is true } \Rightarrow P(a + 3) \text{ is true } \Rightarrow \ldots$$
Theorem 1.1. The sum of the first $n$ positive odd integers is given by:

$$\sum_{i=1}^{n} 2i - 1 = n^2$$
Theorem 1.2. For any real number $r$ except 1, and any integer $n \geq 0$, the sum of the first $n + 1$ elements of a geometric series is given by:

$$
\sum_{i=0}^{n} r^i = \frac{r^{n+1} - 1}{r - 1}
$$
Theorem 1.3. For all integers \( n \) greater than 2,

\[
\prod_{i=2}^{n} \left(1 - \frac{1}{i}\right) = \frac{1}{n}
\]
Theorem 1.4. For all integers $n \geq 0$, $2^n - 1$ is divisible by 3.
Theorem 1.5. For all integers $n \geq 3$, $2n + 1 < 2^n$. 
In class work.

(1) Prove for all integers $n \geq 1$

\[
\sum_{i=1}^{n} 2i = n^2 + n
\]
(2) Prove for all integers $n \geq 0$

$$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$$
(3) Prove for all integers $n \geq 0$ that
\[ \sum_{i=1}^{n+1} i \cdot 2^i = n \cdot 2^{n+2} + 2 \]
(4) Prove for all integers $n \geq 0$ that,

\[ 7^n - 1 \text{ is divisible by 6} \]
(5) Prove for all integers $n \geq 2$ that

$$2^n < (n + 1)!$$