Math/Stat 3850 – Exam 1 Practice

There are 10 questions, worth a total of 100 points.
You may use any calculator, R and any internet resources for this exam, although you are not allowed to ask someone else for help.

(10) 1. R comes with the built in data frame `warpbreaks` containing observations about yarn breakage during weaving.
   (a) What are the variables in this data frame?
   (b) How many cases are there?
   (c) Which is more prone to breakage, type A wool or type B wool?
   (d) Which is most prone to breakage, High, Medium, or Low tension?

   **Solution:** Breaks, wool, tension. 54 cases. Type A. Low tension.

(10) 2. Using the `warpbreaks` data set:
   (a) What are the mean and standard deviation of breaks?

   **Solution:** $\bar{x} = 28.14815$, $s = 13.19864$

   (b) What are the mean and standard deviation of breaks for type B wool?

   **Solution:** $\bar{x} = 25.25926$, $s = 9.300921$

(10) 3. Consider this question:
    If you roll five ordinary dice, what is the probability they add up to exactly 20?
    (a) Use R to simulate this 100,000 times and report the probability estimate you got.

    **Solution:** Between 0.082 and 0.086

    (b) Run the 100,000 trial simulation again. What estimate did you get this time?
    (c) Write your R code here:

    **Solution:** `sum(replicate(100000,sum(sample(1:6,5,replace=TRUE))==20))`

(10) 4. The conversion from Fahrenheit to Celsius is

$$c = \frac{5}{9}(f - 32)$$
Over the last century, August temperatures in St. Louis are approximately normally distributed with mean 77.9°F and standard deviation 2.9°F.

What are the mean and standard deviation of August St. Louis temperatures in °C?

**Solution:** Mean is \( \frac{5}{9}(77.9 - 32) = 25.5 \)°C, standard deviation is \( \frac{5}{9} \cdot 2.9 = 1.61 \)°C.

5. Let the discrete random variable \( X \) have the distribution

\[
P(X = k) = \begin{cases} 0.1 & k = 1 \\ 0.4 & k = 2 \\ 0.3 & k = 6 \\ 0.2 & k = 7 \\ 0 & \text{otherwise} \end{cases}
\]

Compute \( E(X) \).

6. For the random variables described below, name a distribution that would model \( X \) appropriately.

(a) \( X \) is the number of hits on the www.slu.edu website in five minutes.

**Solution:** Poisson

(b) In a World Cup soccer match, \( X \) is the number of goals scored by both teams combined.

**Solution:** Poisson

(c) \( X \) is the total weight of passengers on a fully loaded 747 airplane.

**Solution:** Normal

(d) A camera trap automatically takes pictures of large animals that trigger its sensor. \( X \) is the time that passes between photos.

**Solution:** Exponential

(e) \( X \) is the daily amount of honey produced by a single honeycomb.

**Solution:** Normal
7. The Navy SEALS training course is difficult. Assume each trainee has a 30% chance of completing the course, and that trainees abilities are independent of each other. If the Navy enrolls 200 new trainees, what is the probability that 70 or more will complete the training?

**Solution:** \(1 - \text{pbinom}(69, 200, .3) = 0.07278646\)

8. In the game of craps, players roll two dice and add the numbers shown.
   (a) *The horn* is a roll of 2, 3, 11 or 12. What is the probability of rolling the horn?
   (b) A *natural* is a roll of 7 or 11. What is the probability of rolling a natural?

**Solution:**
- a. \(\frac{6}{36} = \frac{1}{6}\).
- b. \(\frac{8}{36} = \frac{2}{9}\).

9. Let \(X \sim \text{Exponential}(\lambda)\).
   Show that \(P(X > n + 1 \mid X > n)\) does not depend on \(n\), and find its value.

**Solution:**

\[
P(X > n + 1 \mid X > n) = \frac{P(X > n + 1)}{P(X > n)} = \frac{e^{-\lambda(n+1)}}{e^{-\lambda n}} = e^{-\lambda}.
\]

10. Let \(X \sim \text{Uniform}(-1, 1)\) and \(Y \sim \text{Uniform}(-1, 1)\) be two independent random variables. Then \((X, Y)\) is a point on a square. The probability that \((X, Y)\) is inside the unit circle is given by

\[
P(X^2 + Y^2 < 1) = \frac{\text{Area of circle}}{\text{Area of square}} = \frac{\pi}{4} \approx 0.785.
\]

(a) Use R to generate 100,000 values of \(X\) and \(Y\), then estimate \(P(X^2 + Y^2 < 1)\).
   Multiply your result by 4 to get an approximation to \(\pi\).

Write your code here:

```r
x <- runif(100000,-1,1)
y <- runif(100000,-1,1)
4*mean(x^2 + y^2 < 1)
```

(b) Approximate \(P(X^4 + Y^4 < 1)\).

**Solution:** About 0.927.