Problem A:

The -d- system is a formal system with alphabet d and - (hyphen). The system has infinitely many axioms, but all of the same form:

\( xd \) is an axiom, whenever \( x \) is a string of at least two hyphens.

(note that \( x \) must stand for the same string of hyphens in both places).

For example, when \( x \) is --, then --d-- is an axiom.

There is one rule:

From \( xd \), produce \( xd\ y \), where \( x \) and \( y \) consist only of hyphens.

For example, from --d-- , produce --d----

Determine which strings can be produced in this system, by finding an interpretation for the symbols which make it easy to tell if a string can be produced.

Suppose we extend the system with a new symbol c and add a new rule:

From \( xd\ y \), produce \( cx\ y \). where \( x \) and \( y \) are (nonempty) hyphen strings.

Describe the new strings which are produced.

Bonus: Is the string c followed by \( 2^{41} \)-1 hyphens producible?