Exercises

Chapter 4.1  # 5a, 7a
Chapter 4.2  # 11
Chapter 4.3  # 1df, 3df, 5df, 7df

MATLAB/Octave

1. Recall the Lambert W function, defined by \( W(x)e^{W(x)} = x \) and implemented in Matlab as \( \text{lambertw} \). In this problem, you’ll calculate the derivative of \( W \).
   (a) Compute \( W' \) for \( x = [0:.01:5] \) by first computing \( W(x) \) and then using forward divided difference. Time this using Matlab’s \text{tic} \ and \text{toc}. How long did it take?
   (b) Compute \( W' \) by implicit differentiation of \( We^W = x \). Now compute \( W'(x) \) for the same \( x = [0:.01:5] \) using your exact formula for \( W \). How long did this take?
   (c) Plot the error between the exact formula of part (b) and the numeric differentiation of part (a) as a function of \( x \). Where is the error largest? What was the maximum error?

2. Let \( f(x) = x^{1.5} \).
   (a) Numerically compute the second derivative of \( f \) on the interval \([0,4]\) by taking the first derivative twice. You can use forward or backward differences as you wish. Compare these values with the correct values (given by symbolically differentiating \( f \)). Make a plot of the error between the approximated and exact values.
   (b) Repeat part (a) but use the second derivative midpoint formula to compute the second derivative

3. Use Matlab to draw the spline \( S \) with nodes 1, 2, 3, 4, 5, 6, 7, 8 and values 3, 1, 4, 1, 5, 9, 2, 6. Now plot the derivative, second derivative, and third derivative of \( S \). Describe what you see, and explain.