Math 320  Homework 5  Due Wednesday, March 19

Read BF Chapter 3.3, 3.5

Exercises

**Chapter 3.3**  # 1*, 3*, 7a, 10, 11, 16, 17
* You should know how to do this sort of problem, but I recommend you do A, B, C below and then skip 1 and 3, since the numbers in 1 and 3 are horrible.

**Problem A:** Find the Newton form of the fourth degree polynomial interpolating $n!$ at $n = 0, 1, 2, 3, 4$.

**Problem B:** This example was first computed by James Gregory in 1670. The Newton forward-difference formula is sometimes known as Newton-Gregory interpolation.

Use the forward difference method to interpolate $f(x) = x^3$ at $x = 10, 15, 20, 25, 30$ and use it to compute $23^3$.

**Problem C:** Compute $\ln 1.09$ correct to five decimal places by interpolating natural logarithms for $1, 1.1, 1.2, 1.3$.

---

**Chapter 3.5**  # 3a, 13

**Problem D:** Compute the natural cubic spline through $(0, 0), (1, 10)$, and $(2, 4)$.

**Problem E:** Explain why the following function is not a spline:

$$f(x) = \begin{cases} 
-x^2 + 1 & \text{if } 0 \leq x \leq 1 \\
(x - 1)^3 + 2(x - 1) & \text{if } 1 \leq x \leq 2 \\
2(x - 2) + 3 & \text{if } 2 \leq x \leq 3 
\end{cases}$$
MATLAB/Octave

1. In this question, you investigate the interpolating polynomial for \( f(x) = \frac{1}{x} \) with nodes at \( 1, 2, \ldots, n \).
   (a) Find (by hand, with exact fractions) the Newton form of the interpolating polynomial for \( f(x) = \frac{1}{x} \) with nodes at 1, 2, 3, 4. Repeat with nodes 1, 2, 3, 4, 5.
   (b) (Optional, challenging) Prove the interpolating polynomial for \( \frac{1}{x} \) with nodes 1, 2, 3, \ldots, \( n \) takes the form:

   \[
   p_n(x) = 1 - \frac{1}{2!}(x - 1) + \frac{1}{3!}(x - 1)(x - 2) - \frac{1}{4!}(x - 1)(x - 2)(x - 3) + \cdots + \frac{1}{n!}(x - 1)(x - 2) \cdots (x - (n - 1))
   \]

   (c) From part (b), rewrite \( p_n(x) \) in the nested form:

   \[
   p_n(x) = 1 - \frac{1}{2}(x - 1) \left[ 1 - \frac{1}{3}(x - 2) \left[ 1 - \frac{1}{4}(x - 3) \left[ \cdots \frac{1}{n}(x - (n - 1)) \right] \cdots \right] \right]
   \]

   Write a Matlab function \texttt{harmonicpoly(n,x)} that computes \( p_n(x) \) using the nested form.
   (d) The interpolating polynomial \( p_n(x) \) converges to \( \frac{1}{x} \) as \( n \to \infty \). However, it makes a big difference how you compute it:
      i. Use \texttt{polyinterp} to graph \( p_n(x) \) for \( x \in [1, 4] \). Increase \( n \) until it starts to go wrong.
      ii. Use \texttt{newtoninterp} to graph \( p_n(x) \) for \( x \in [1, 4] \). Increase \( n \) until it starts to go wrong.
      iii. Use \texttt{harmonicpoly} to graph \( p_n(x) \) for \( x \in [1, 4] \). Increase \( n \), it should stay near \( \frac{1}{x} \).

   For this part, you can write down the values of \( n \) at which things break down, and describe what each graph does. Or, print out pictures of the graphs when they begin to fail.

2. Using Matlab’s \texttt{spline} function, create a script that draws a (rough) picture of the Billiken.