Math 320 – Review Questions

1. The polynomial \( \chi(t) = t^7 - 12t^6 + 67t^5 - 230t^4 + 529t^3 - 814t^2 + 775t - 352 \) is important in graph theory.
   (a) Write this polynomial in nested form.
   (b) How many multiplications are needed to compute \( \chi(t) \) using Horner’s algorithm?

2. The “logarithmic integral” function \( li \) is defined by \( li(x) = \int_0^x \frac{dt}{\ln t} \).
   (a) Given that \( li(4) = 2.96759 \) and \( li(5) = 3.63459 \), use linear interpolation to compute an approximation to \( li(4.3) \).
   (b) Give a good bound on the error in your approximation from part (a).
      (Hint: By the fundamental theorem of calculus, \( \frac{d}{dx} li(x) = \frac{1}{\ln x} \))

3. Find a cubic polynomial \( p \) that takes all of the following values:

\[
\begin{array}{c|c|c|c|c|c}
   x  & 0  & 1  & 2  & 3  & 4  \\
   p(x) & -82 & -19 & 0 & -1 & 233 \\
\end{array}
\]

4. A smooth function \( f \) has values

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.2</td>
<td>0.67474</td>
</tr>
<tr>
<td>-0.1</td>
<td>0.73282</td>
</tr>
<tr>
<td>0</td>
<td>0.78540</td>
</tr>
<tr>
<td>0.1</td>
<td>0.83298</td>
</tr>
<tr>
<td>0.2</td>
<td>0.87606</td>
</tr>
</tbody>
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Compute \( f'(0) \) as well as you can. Explain your method.

5. We used a MATLAB function \( \text{polyinterp}(x,y,u) \) to compute \( P(u) \) where \( P \) is the interpolating polynomial with \( P(x_i) = y_i \).
   (a) Write a MATLAB command (or commands) make a plot of \( P \) on the interval \([-8, 8]\), where \( P \) interpolates \( \arctan(x) \) for \( x = -8, -7, -6, -5, \ldots, 5, 6, 7, 8 \).
   (b) The result of part (a) would look like this:

   Explain the picture.