Reading

- Borwein\textsuperscript{2}, “On the history of the calculation of π”.
- Stillwell, Chapter 9.5-9.7, 10-10.3
- Berkeley, \textit{The Analyst}, sections I-XX, XLIV-XLVI
- Brown, “The fundamental anagram of calculus”.

Exercises

Stillwell

- 9.5.3, 9.5.4 (binomial series and \(\sin^{-1}\))

Borwein\textsuperscript{2}

Exercise 4.

Hints: For the first part, complete the square and make a trig substitution (or use the arcsin integral). For the second part, use Newton’s binomial series on page 169 of Stillwell and then integrate term-by-term.

Challenge: For Newton to compute \(\pi\) to 15 digits, how many terms of the series would he have needed?

Logarithms

In these problems \(\log\) is the natural logarithm.

1. Show that

\[
\log(2) = 2\log(1.2) - \log(0.8) - \log(0.9)
\]

and

\[
\log(3) = \log(1.2) + \log(2) - \log(0.8).
\]

2. Use the power series for \(\log(1 + x)\) to compute \(\log(1.2), \log(0.9),\) and \(\log(0.8)\) to at least five decimal places, and then compute \(\log(2)\) and \(\log(3)\). Newton did this by hand, and it’s not too hard. Use a calculator if you must.

3. Use the Gregory-Newton interpolation formula (Stillwell pg 186) with \(a = 2\) and \(b = 1\) to compute an approximation to \(\log(2.5)\). Use just the \(\Delta\) term first, then compute \(\log(4) = 2\log(2)\) and use the \(\Delta^2\) term as well. Compare your result with the correct value \(\log(2.5) \approx 0.91629\).

Some History

1. What was the purpose of the anagram in Newton’s 1677 letter to Leibniz?
2. What is Berkeley struggling to understand in \textit{The Analyst} sections XLIV-XLVI?
3. What is the Lucasian Chair of mathematics? Who currently holds it?

More to come...?