Reading

- Joseph, pages 461 (bottom) - 487
- Stillwell, Chapter 6 (except we’re not covering 6.2 and 6.6)
- MacTutor History of Mathematics archive, *Tartaglia versus Cardan*

Exercises

Lattice Multiplication

Compute $1990 \times 365$ using lattice multiplication.

Compute $(x^2 + 3x - 1)(2x^2 - x + 5)$ using lattice multiplication.

Quadratic Equation

Page 478 of Joseph has a diagram of al-Khwarizimi’s geometric solution to the quadratic $x^2 + 10x = 39$. Make up another example and draw the corresponding diagram.

Fibonacci Numbers

Assuming $\lim_{n \to \infty} \frac{F_{n+1}}{F_n}$ exists, prove it is the golden ratio $\frac{1 + \sqrt{5}}{2}$.

Polynomial Equations

1. Solve $x^3 + 60x^2 + 1200x = 4000$. Hint: complete the cube. (Dardi of Pisa, 1344)
2. Antonio de Mazzinghi (1353-1383): Find two numbers such that multiplying one by the other makes eight and the sum of their squares is twenty-seven. Solve this in two ways: 1) By eliminating one variable and solving the resulting fourth degree equation with the quadratic equation. 2) using de Mazzinghi’s technique of setting the two numbers to $a + \sqrt{b}$ and $a - \sqrt{b}$. Check numerically that your two answers are the same.
3. Express $\sqrt{27 + \sqrt{200}}$ as $a + \sqrt{b}$ for integer $a$ and $b$. (From *Coss*, by Christoph Rudolff. This 1525 book introduced the notation $\sqrt[3]{}$ for roots.)

Cubic Equations

Give exact answers to all of these, then find a decimal approximation.

1. Solve $x^3 + 24x = 16$.
2. Solve $x^3 + 27x = 6x^2 + 58$.
3. Solve the problem that Zuanne da Coi proposed to Tartaglia in 1530: Find a number which multiplied by its root increased by three equals five.
4. Fior’s first question to Tartaglia in the challenge of 1535: Find the number when added to its cube root gives six.
5. Tartaglia’s second question to Fior in the challenge of 1535: Find an irrational quantity such that when it is multiplied by its square root diminished by thirty, the result is a given rational number.
6. Write one (or more) scathing insults you would cast upon a mathematician who cannot solve cubic equations. Bonus points for period language.