Reading

- Stillwell, Chapter 3, 5.1, 5.3-5.5, 5.7 (note we aren’t using 3.4, 3.5, 5.4, 5.5 yet)
- Joseph, pages 380-392

Exercises

Formulas For Primes

1. (Euler’s polynomial, 1772) Is it true that \( p(x) = x^2 + x + 41 \) is prime for every \( x = 0, 1, 2, \ldots \)?

2. Show that the Mersenne number \( M_n = 2^n - 1 \) is composite when \( n \) is composite, so that the only possible Mersenne primes are \( M_p \) for \( p \) prime.
   (Hint: Write \( n = ab \) and \( M_n = 2^{ab} - 1 = (2^a)^b - 1^b \)

GCD and Linear Diophantine Equations

1. Stillwell: # 5.3.1, 5.3.2

2. For each of these, decide if it can be solved. If so, find any solution:
   (a) \( 24x + 138y = 18 \)
   (b) \( 14x + 35y = 87 \)
   (c) \( 221x + 35y = 11 \)

3. Suppose that \( x_0, y_0 \) is a solution to the Diophantine equation \( ax + by = c \). Assume that \( \gcd(a, b) = 1 \), and show that all solutions to the equation are given by \( x = x_0 + bt, \ y = y_0 - at \) for integer values of \( t \).

4. (Alcuin of York, 775AD) A hundred bushels of grain are distributed among 100 persons in such a way that each man receives 3 bushels, each woman 2 bushels, and each child \( \frac{1}{2} \) bushel. How many men, women, and children are there?

5. (Mahaviracarya, 850AD) There were 63 equal piles of plantain fruit and 7 single fruits. They were divided evenly among 23 travelers. What is the number of fruits in each pile?