Discrete Math – Take Home Quiz 1

This quiz should take you approximately 25 minutes. You may use reference material, but are not allowed to ask for help from anyone except Dr. Clair.

(10) 1. Show that \((q \lor s) \land (r \lor p) \land (\neg s \lor \neg p) \land (s \lor \neg q)\) is satisfiable.

(10) 2. Suppose \(x\) and \(y\) are integers. True or false:

(a) \(\forall x (x^2 \geq x)\)

(b) \(\exists x (x^2 \leq x)\)

(c) \(\exists ! x (x^2 = 9)\)

(d) \(\forall x (x^2 > 0 \rightarrow x > 0)\)

(e) \(\forall x \exists y (x = y^2)\)

(10) 3. In the Venn diagrams below, \(A\), \(B\), and \(C\) are the three circular regions.

Describe each shaded set in terms of \(A\), \(B\), \(C\) and basic set operations.
4. Suppose $a, b, c$ are positive real numbers. 
Prove that if $abc > 1000$ then one of $a$, $b$, or $c$ is larger than 10.

5. Suppose $x, y \in \mathbb{Z}$ with $x^2 = 1 + 13y^2$. Show that exactly one of $x$ and $y$ must be odd.
Extra credit: Find an example of $x$ and $y$. 