1. Suppose $M$ is a manifold and $g, h$ are both Riemannian metrics on $M$. Show that for any $p \in M$, there is a neighborhood $U$ of $p$ and scalars $\lambda, \mu > 0$ so that for any $x \in U$ and any $v \in T_x M$, we have:

$$\lambda g(v, v) \leq h(v, v) \leq \mu g(v, v).$$

2. Let $\omega$ be an arbitrary smooth 1-form on $M$. For vector fields $X, Y \in \mathfrak{X}(M)$, define $\Omega(X, Y) = \omega([X, Y])$. Show that $\Omega$ is bilinear and anti-symmetric. Is $\Omega$ a tensor?

3. Given intervals $I, J$, and unit speed curves $f : I \to M$, $g : J \to M$, let $\Gamma \subset I \times J$ be the set of $(s, t)$ with $f(s) = g(t)$. Because $g^{-1} \circ f$ is a diffeomorphism on its domain, $\Gamma$ consists of line segments of slope $\pm 1$ which must extend to the boundary of $I \times J$, and at most one of these segments can end on a given edge of the rectangle $I \times J$. Explain why $\Gamma$ has at most two components, and draw all combinatorial possibilities for $\Gamma$. 