- Boothby pg 187 #1 is a straightforward warm-up problem with basic linear algebra facts about bilinear forms.

- Boothby pg 187 #2: Show there is a correspondence:

  fields of bilinear forms on $M \leftrightarrow C^\infty(M)$-bilinear mappings $\mathcal{X}(M) \times \mathcal{X}(M) \to C^\infty(M)$

Hints: The $\rightarrow$ direction is easy. In the other direction, you have a bilinear mapping of vector fields on $M$, and given $v, w \in T_p(M)$, you need to define $\Phi_p(v, w)$. Do this by extending $v, w$ to vector fields, and then proving the definition is independent of the extensions chosen. The key step is to use local coordinates near $p$. If two extensions agree at $p$, then their coefficients in coordinates agree at $p$. Applying a cutoff function, you can extend the coefficients to all of $M$ and pull them out of $\Phi$.

Another big hint is that this is the specific case of Lee’s Proposition 7.32, with $r = 0$ and $s = 2$.

- Boothby pg 192 #3 (definition of the gradient)

1. Given $c(u) = (r(u), z(u))$ a smooth curve in the $x$-$z$ plane with $r(u) \neq 0$, let $M \subset \mathbb{R}^3$ be the surface of revolution of $c$ around the $z$-axis. Find the metric $g$ on $M$ as a submanifold of $\mathbb{R}^3$.

- Boothby pg 192 #2 (the metric on the torus in $\mathbb{R}^3$) You might apply the previous problem.