1. Lee Exercise 1.118. Given smooth \( f : \mathbb{R}^n \to \mathbb{R}^m \), show the graph of \( f \) is a regular submanifold of \( \mathbb{R}^m \times \mathbb{R}^n \).

2. Lee Exercise 1.119. The main point of this exercise is to show that every regular submanifold \( M \) of \( \mathbb{R}^n \) is locally the graph of a function.

3. For a real number \( a \), define \( f : \mathbb{R}^2 \to \mathbb{R} \) by \( f(x, y) = x^3 - 3ax - y^2 \). Find all values of \( b \) so that \( f^{-1}(b) \) is a manifold. Graph \( f^{-1}(b) \) for a variety of \( a \) and \( b \), including the critical values of \( b \). These manifolds are called elliptic curves.

4. Lee Ch 3 Problem 28. For a homogeneous polynomial \( p \) of \( n \) variables, show \( p^{-1}(c) \) is a submanifold of \( \mathbb{R}^n \) for all \( c \neq 0 \).

5. Lee Ch 3 Problem 3. Show if \( M \) is compact and \( N \) is connected, then a submersion \( f : M \to N \) must be surjective.

6. Define the map \( H : \mathbb{R}^4 \to \mathbb{R}^3 \) by
\[
H(x, y, z, w) = (2(xy + zw), 2(xw - yz), x^2 + z^2 - y^2 - w^2).
\]
Check that restricting \( H \) to \( S^3 \subset \mathbb{R}^4 \) defines a map from \( S^3 \) to \( S^2 \), the Hopf map. Show that the Hopf map is a submersion. What are the fibers of the Hopf map (i.e. what manifold is \( f^{-1}(q) \) for \( q \in S^2 \))?